

# Two years later—lessons from vortex dynamics in super media

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## Abstract

Vortex dynamics in super media has been one of most intriguing fields in physics. It is therefore not surprising that it had troubled great physicists such as L. Landau. 50 years after its conception, in 1990's the consistent fundamental vortex dynamics equation emerged after long and serious effort by a group of researchers associated with Seattle. Nevertheless, this fundamental equation has been met with resistance. Two years ago the present author analyzed this situation (cond-mat/0407007). Five "mistakes" were identified to explain this resistance. Given the current tremendous interest in vortex dynamics, it would be desirable to provide a progress report: A survey of literature reveals that 3 out 5 "mistakes" has in fact been confirmed by other researchers, not a bad appreciation.

## I. INTRODUCTION

To understand the resistance in the acceptance of the fundamental equation of vortex dynamics in super media, five "mistakes" were identified two years ago. They are:

- 1) Misuse of the relaxation time approximation;
- 2) Mixing with another effect;
- 3) Double counting the same topological effect;
- 4) No extra Berry phase contribution at vortex core; and
- 5) Hall anomaly and vortex many body effect.

For background material, the fundamental contributions went back to Bardeen and Stephen [1] and to Nozieres and Vinen [2]. The first direct measurement of the transverse force on moving vortices in superconductors was reported in 1997 in Sweden [3]. More background materials can be found in cond-mat [4, 5, 6].

It should be kept in mind here that we are talking about the "mistakes" made by a very capable group of theoretical physicists. Normally it is already rare to make one such "mistake" by any of them. Instead, five have been identified collectively. To some, it would be possible that there could exist mistake(s) in the identification process. Thus, it is reasonable to ask how it goes after two years.

A survey of recent literature has shown that till now three such mistakes have been actually confirmed during past two years by other researchers. Given the amount of intellectual effort, this is a rather impressive number: 3 out 5, already a majority. In the following we discuss them one by one.

## II. MISUSE OF THE RELAXATION TIME APPROXIMATION

The most subtle inconsistency is the relaxation time approximation employed by Kopnin and his co-workers (Kopnin and Kratsov, JETP Lett., 1976; Kopnin and Lopatin, Phys. Rev. B, 1995; van Otterlo, Feigelman, Geshkenbein, Blatter, Phys. Rev. Lett., 1995, see the discussions and references in Ref's. [5, 6]). This type of mistakes frequently occurs in the force-balance type calculation of transport coefficients, already noticed at least by Green in the 1940's and extensively and repeatedly discussed by R. Kubo. Unfortunately, as R. Kubo remarked in his coauthored famous book on statistical physics, in the literature such error

repeatedly appears in different disguises.

Expounding a statement in Thouless, Ao, and Niu [7], it was explicitly formulated that vortex friction can be obtained without such relaxation time approximation [8]. The Bardeen-Stephen case with disorder for s-wave superconductors was then explicitly verified within this formulation [8]. There is no influence on Magnus force, expected from Anderson's dirty superconductor theorem and the topology (Berry phase, spectral flow, chiral symmetry, etc).

Now, this formulation [8] has been further extended to the d-wave superconductors [9]. From the point of vortex core contribution to vortex friction, this is the first nontrivial result in more than 40 years since Bardeen-Stephen [1].

### III. MIXING WITH ANOTHER EFFECT

The second type of mistake is very interesting. It concerns the contribution of "phonons" (Sonin, Soviet Phys. JETP, 1976; Phys. Rev. B, 1997, see the discussions and references in Ref's. [5, 6]).

The problem is clear: At zero temperature according to Sonin there should not exist any effect. According to a group of researchers loosely associated with Seattle, there should be an dissipative effect at zero temperature: Zero for constant vortex velocity but finite when vortex accelerating [10, 11, 12]. This is a non-conventional damping, corresponding to the non-ohmic damping in the quantum dissipative dynamics studied by Leggett.

Now, such effect has been suggested to be responsible to account for the dissipation puzzle in quantum turbulence at zero temperature [13], closely resembles to the process discussed by Vinen [14].

### IV. HALL ANOMALY AND VORTEX MANY BODY EFFECT

This is the so-called Hall anomaly: How could the large and universal transverse force (Magnus force) lead to a small Hall angle, and, sometime to a sign change? The ingredients to explain the Hall anomaly are actually already implied in the works of Dirac and Abrikosov.

Let us stand one step away from vortex dynamics and consider the Hall effect in semiconductors. The transverse force there, the Lorentz force, on a moving electron is universal.

There exist such extremely rich Hall phenomena in semiconductors: small Hall angle, sign change, Quantum Hall effect, etc. It would be puzzling that how could a universal Lorentz force generate such a complexity. Fortunately, this puzzle has long been solved: The competition between electron many body effect (Coulomb interaction, Fermi-Dirac statistics) and pinning (lattice, impurities, etc). The key to solve the puzzle is logically parallel to Dirac's idea of a void in a filled Fermi sea: the existence of holes in a filled energy band.

The ubiquitous existence of the Abrikosov lattice, the starting point of theoretical considerations in the mixed state of superconductors, already loudly suggests that one must consider vortex many body effect. Following this suggestion, it was argued in 1995 that the competition between vortex many body interaction and pinning can explain the Hall anomaly in the mixed state [15]. It may be further pointed out that the vortex many body effect-pinning model appears consistent with all major experimental observations on Hall anomaly [16]. The earlier theories by Kopnin and his associates/collaborators on Hall anomaly based on independent vortex dynamics model have been shown to be wrong both mathematically [17] and physically [18].

Because of those critiques on their work, Kopnin and Vinokur later published a paper [19] completely embraced the idea of vortex many body plus pinning for Hall anomaly. The interesting thing is that they completely "forgot" the relevant prior theoretical work. It may be instructive to compare the abstract of Ao, 1998 [16]:

*Physical arguments are presented to show that the Hall anomaly is an effect of the vortex many-body correlation rather than that of an individual vortex. Quantitatively, the characteristic energy scale in the problem, the vortex vacancy formation energy, is obtained for thin films. At low temperatures a scaling relation between the Hall and longitudinal resistivities is found, with the power depending on sample details. Near the superconducting transition temperature and for small magnetic fields the Hall conductivity is found to be proportional to the inverse of the magnetic field and to the quadratic of the difference between the measured and the transition temperatures.*

to the abstract of Kopnin and Vinokur, 1999 [19]:

*We demonstrate that pinning strongly renormalizes both longitudinal and Hall resistivity in the flux flow regime. Using a simple model for the pinning potential we show that the magnitude of the vortex contribution to the Hall voltage decreases with increase in the pinning strength. The Hall resistivity  $\rho_{xy}$  scales as  $\rho_{xx}(2)$  only for a weak pinning. On the*

*contrary, a strong pinning breaks the scaling relation and can even result in a sign reversal of  $\rho(xy)$ .*

The readers are advised to read the full original articles in the span of 10 years. It is clear that once the correct physical understanding has been reached, there exists numerical other mathematical approaches.

During past two years, explicit and controllable experiment was designed to test the idea of vortex many body effect for Hall anomaly. The experimental results were surprisingly consistent with theoretical predictions [20].

We also witness further development of theory based on this vortex many-body effect plus pinning for Hall anomaly from the competing research group [21].

Because same results have been repeatedly obtained by competing groups it may be concluded that the Hall anomaly is now solved at least conceptually. This is not an easy achievement given its vortex-like history bothering at least two generations of condensed matter physicists.

## V. FURTHER PROGRESS

Though 3 out of 5 have been confirmed by other researchers, it is still possible in the remaining 2 "mistakes" the present author identified could be mistakes in themselves. Because nobody can prediction the future, we have to wait for the further progresses on vortex dynamics. The only thing we can hope is that this would be too long. In the same time, it is reasonable to expect that the three mistakes already identified will continue to be identified in one form or another by more researchers.

*The reference list here is incomplete. A balance reference list was attempted in [6]. Any suggestion on interesting references will be appreciated.*

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